Determination Of The Bulk Helium Critical Exponents Using Confined Helium

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Abstract. The specific heat of helium homogeneously confined in one or more dimensions is expected to collapse onto a scaling function which depends only on the ratio of the smallest dimension of confinement to the correlation length, written as L/ξ. This may be rewritten to explicitly show the temperature dependence of the correlation length as $L/\xi_0 t^{-\nu}$, where the constant ξ_0 is the prefactor of the correlation length, t is a dimensionless temperature difference from the superfluid transition, and ν is the critical exponent associated with the correlation length. Thus, in principle, one should be able to obtain the exponent ν from the scaling of thermodynamic measurements of confined helium for various L's. This would represent an independent determination of ν distinct from what is obtained using the behavior of the bulk superfluid density, or via the bulk specific heat and the hyperscaling relation. In practice, this analysis is hampered by the lack of a theoretical expression for the scaling function. We present preliminary results of analyses of specific heat data for planar confinement which spans a range of about 1200 in L and yields the exponent ν. The data are fit to an empirical equation which is obtained so that it has the proper asymptotic temperature dependence for large and small values of the scaling variable, which we take as $tL^{1/\nu}$. Results are compared with theoretical and other experimental determinations of ν.

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Finite-size scaling theory states the specific heat of confined helium will scale as a function of $(L/\xi)^{1/\nu}$ where L is the spatial confinement, ξ is the correlation length, and ν is the bulk correlation-length critical exponent [1, 2]. Typically, this exponent is determined by a thermodynamic measurement such as the superfluid density [3] or derived from the specific heat [4] of bulk helium. However, the reverse may be done; one may ask which exponent provides the best collapse of specific heat data measured for a number of planar confinements.

In this analysis, we use specific heat data from measurements of helium confined to eight different planar confinements. The seven smallest confinements [5–7], which range from 48.3 nm to 986.9 nm in L, may be measured on Earth since effects due to a pressure gradient across the sample are not evident within the temperature resolution of the experiments. The largest confinement, which has a spatial separation between its plates of 57000 nm, was measured in a near-Earth orbit to reduce the effects of the pressure gradient across the thick slab of helium [8, 9]. The full range of confinements is nearly a factor of 1200 from smallest to largest. Figure 1 shows the $T > T_\lambda$ specific heat measured for ^{4}He confined in the eight different planar confinements. To attempt to scale the data, one uses

$$[C_p(\infty,t) - C_p(L,t)]\, t^{\alpha} = g_2(y), \tag{1}$$

where $y = tL^{1/\nu}$ and $t = T/T_\lambda - 1$. α is the critical exponent of the specific heat. Once the specific heat data is

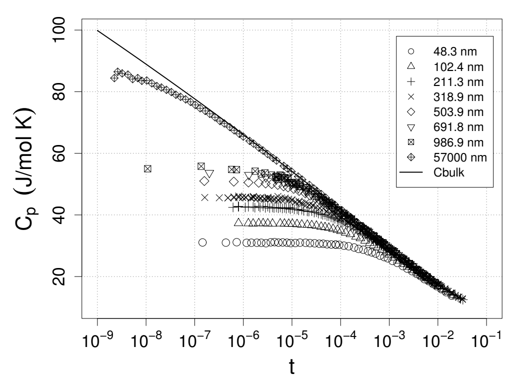

FIGURE 1. Specific heat data from helium confined to eight different planar geometries. This is for temperatures greater than T_λ.

cast in scaling form using Eq. 1, one needs a specific expression to use in fitting the data to determine the goodness of collapse for a given ν. Since there is not a known theoretical expression for the scaling function, we use an empirically-derived function chosen to have the proper limits for large and small values of the dimensionless scaling variable, $t\,(L/a_0))^{1/\nu}$ where a_0 is 3.56 Å. This is given by [10]

$$g_2(y) = \frac{A/\alpha}{1+ay^{\nu}} + \frac{by^{\alpha}}{1+cy^{\alpha+\nu}}. \tag{2}$$

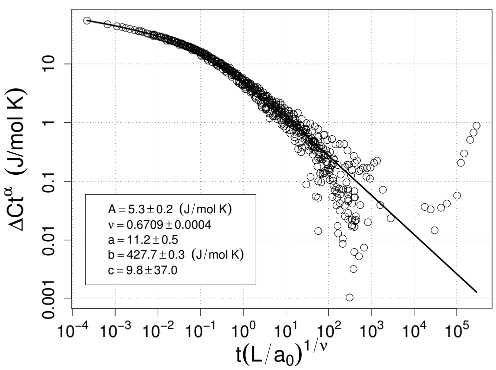

FIGURE 2. Best collapse of the scaled data. The solid line is described by Eq. 2 and the best collapse parameters shown on this plot.

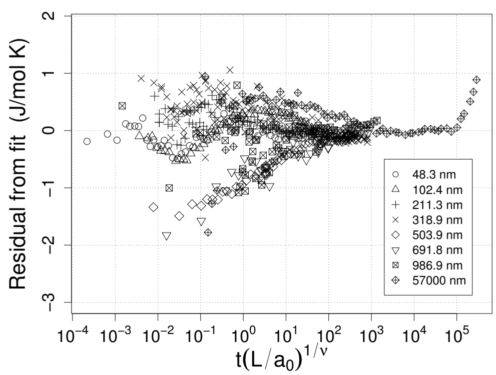

FIGURE 3. Residuals between the solid line and the scaled data in Fig. 2.

The two critical exponents are related by the hyperscaling equation

$$\alpha = 2 - 3\nu, \tag{3}$$

therefore Eq. 2 may be rewritten using only a single critical exponent as a fit parameter. A is the leading amplitude of the specific heat, and along with ν, a, b, and c is varied to best fit ΔC to $t^{(2-3\nu)}g_2(y)$. This procedure yields $\nu = 0.6709 \pm 0.0004$. This compares to 0.67155 ± 0.00027, 0.6709 ± 0.0001, and 0.6705 ± 0.0006, which correspond to the latest theoretical estimate [11], the latest value derived from Eq. 3 and the bulk specific heat [4], and the latest value from the superfluid density of bulk helium [3] respectively. *We find the experimental results remarkably close considering the variety of experiments used to determine* ν. The scaled data plotted using the exponent which provides the best collapse are shown in Fig. 2. Figure 3 shows the difference between this data and the line described by Eq. 2 and the best collapse parameters.

Ultimately, our analysis relies upon the function used to perform the least-squares minimization of the data in scaling form. It is certain that another function would provide different values for the fit parameters, including ν. Ideally, a method of determining the best collapse without relying on a specific function should be used. One method we have considered is to scale the data with a fixed ν and divide this data into a fixed number of equal-width bins based upon the scaling variable. We then would perform a linear regression fit of the data within each bin and record the residual error of the fit. The sum of the errors for all bins would be the measure of the goodness of collapse for a particular ν. We next repeat the procedure using different values for ν until we determine a neighborhood of values for ν which provide the smallest total residual error indicating the best collapse. While this will predict the value of ν which provides the best collapse, factors such as method used to weight the data and number of bins affect the result. We will continue to explore this method as a means to determine ν using confined helium.

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