

Testing The Universality Of The Lambda Transition Using Confined Helium Mixtures

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Abstract. The universality of phase transitions is an important prediction of theories of critical behavior. Simply stated, microscopically different systems near a critical point may be described by universal quantities if their dimensionality is the same and the order parameter has the same degrees of freedom. One way to test this idea is to measure the thermodynamic response of a set of systems to an input where the response changes with a variation in some quantity like spatial confinement, impurity concentration, or even pressure. While the response of each system is different, the behavior may still be described by a common critical exponent if the idea of universality is correct. Confined mixtures of ^3He - ^4He and pure ^4He are believed to satisfy these requirements. Here, amplitudes such as the magnitude of the correlation length and the temperature of the transition both depend upon the concentration of the mixture and the extent of confinement. However, universality predicts the value of the critical exponent that describes the behavior near the transition should not be affected. Therefore, mixtures confined to a planar film should scale with the same critical exponent as pure ^4He , regardless of concentration or magnitude of film thickness. We used two different planar confinement cells: one at 48.3 nm and a second at 986.9 nm. We compare specific heat data taken from seven different concentrations split between the two cells and use finite-size scaling theory in an attempt to collapse all the data onto a universal curve using a single critical exponent.

Keywords: helium, mixtures, scaling, specific heat, universality

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The superfluid transition of helium is an example of a continuous phase transition. Near the transition temperature, thermodynamic fluctuations dictate the behavior of the free energy. This transition has been studied quite extensively and the specific heat and superfluid density are well known in the thermodynamic limit. The behavior of helium confined to be homogeneously small in one dimension has also been studied [1–3]. Here, finite-size effects become evident near the transition since the temperature-dependent correlation length becomes comparable to the spatial confinement. Less well studied is the behavior of confined mixtures of ^3He - ^4He . Compared to ^4He , mixtures have a larger correlation length and lower transition temperature. This reduction in transition temperature is a function of concentration and defines a locus of transitions known as the λ -line. Mixtures are expected to belong to the same universality class as pure helium and therefore should be described by the same critical exponents. Thus, mixtures of ^3He - ^4He may be used to test the ideas of universality along a line of continuous transitions.

The cells used to confine helium to a planar geometry have been discussed in detail elsewhere [1, 2]. Briefly, these cells consist of two silicon wafers directly bonded together. One wafer has a particular thickness of oxide grown on it. The majority of this oxide is removed leaving a 4 mm wide ring on the outer circumference and an array of 0.2 mm square posts inside this ring. Upon bonding, the two wafers and border form a closed cell with the

border and posts separating the two wafers by a distance defined by the oxide thickness. Helium is introduced into the cell through a hole in one of the wafers.

The small sample size requires the use of an ac technique to measure the heat capacity [4, 5]. A periodic voltage is applied to a heater on the cell bottom. This produces temperature oscillations which are measured using thermometers affixed on top of the cell. The heat capacity is related to the magnitude of the heat input and resulting temperature oscillations.

Since ^3He is a mobile impurity in the system, the heat capacity measured at constant pressure and concentration is renormalized and is not expected to show true critical behavior [6]. Therefore, one must perform a conversion to a specific heat at constant pressure and difference in chemical potential between ^3He and ^4He , $\phi = \mu_3 - \mu_4$. The conversion involves derivatives taken along the lambda line and is given by [7, 8]

$$C_{p\phi} = \frac{C_{px} \left(T \frac{\partial x}{\partial T} \Big|_{p,t} \frac{\partial \phi}{\partial T} \Big|_{p,t} - T \frac{\partial s}{\partial T} \Big|_{p,t} \right) + \left(T \frac{\partial s}{\partial T} \Big|_{p,t} \right)^2}{T \frac{\partial s}{\partial T} \Big|_{p,t} + T \frac{\partial x}{\partial T} \Big|_{p,t} \frac{\partial \phi}{\partial T} \Big|_{p,t} - C_{px}} \quad (1)$$

where s is the molar entropy and the derivatives in this conversion are tabulated in [7].

Finite-size scaling theory [9, 10] predicts that the heat capacity of confined helium, $C_{p\phi}(L, \theta)$, will scale as a

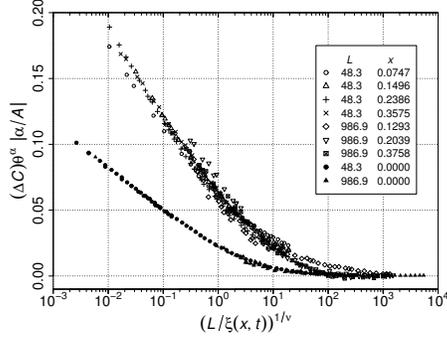


FIGURE 1. Here, all data are scaled using an α determined from each particular concentration.

function of L/ξ and is given by

$$\Delta C \theta^\alpha \left(\frac{\alpha}{A(x)} \right) = g_2 \left(\left[\frac{L}{\xi(x, \theta)} \right]^{1/\nu} \right). \quad (2)$$

Here, $\Delta C = [C_{p\phi}(\infty, \theta) - C_{p\phi}(L, \theta)]$, $\theta = |T - T_\lambda|/T_\lambda$ is a dimensionless reduced temperature appropriate for $C_{p\phi}$ [7], L is the spatial confinement length, ξ is the correlation length, $A(x)$ is the amplitude of the bulk specific heat, and α and ν are the critical exponents associated with the specific heat and correlation length respectively. The ratio $(\alpha/A(x))$ removes the leading concentration-dependent amplitude of the heat capacity and allows the mixtures and pure system to be compared on the same scale.

Figure 1 shows 9 sets of data for $T > T_\lambda$ scaled according to equation 2. Here, the α used for each mixture in the scaling equation is unique and comes from fitting bulk data at each concentration. While the mixture data collapse, it is obvious that the pure data (solid symbols) stand apart. This is due to the fact that the bulk mixture data yield an exponent α which is close to -0.025 , a factor of ~ 2 more negative than for pure ^4He .

Another analysis is shown in figure 2. Here, data are scaled using an universal $\alpha = -0.0115$ derived from superfluid density measurements of pure ^4He [11]. For this analysis, one must assume the exponent which is determined from fitting the bulk $C_{p\phi}(x)$ is an effective exponent and the proper exponent used to scale the data is the one determined from the $x = 0$ data. However, the amplitudes $A(x)$ come from fits of the data where α is allowed to take on its best fit value, ~ -0.025 . While the collapse of all the data in Fig. 2 is perhaps satisfactory, one must remember this is not a self-consistent analysis since the value of α used to scale the data is not the same as that used to determine the magnitude $A(x)$ of the specific heat of the mixtures.

In summary, we have tested the ideas of universality using confined mixtures of helium. This includes seven

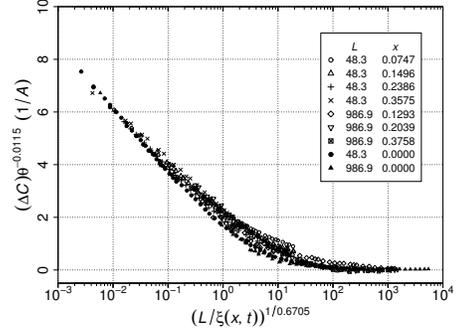


FIGURE 2. Scaled data for seven concentrations measured in two different confinement cells at $T > T_\lambda$. Here, the same α is used for the pure system and the mixtures.

different concentrations split between two different spatial confinements. Data for $x \neq 0$ scale amongst themselves but there is not a self-consistent means to allow the $x = 0$ to scale with the $x \neq 0$ data. One may question if these results indicate that the point at $x = 0$ on the phase diagram is special. This would be consistent with the determination of a different exponent obtained for bulk pure ^4He and mixtures of ^3He - ^4He . However, the uniqueness of this point is not expected on theoretical grounds.

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