

1D Crossover, Universality and Finite-Size Scaling of the Specific Heat

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Abstract.

We report measurements of the specific heat of ^3He - ^4He mixtures near the superfluid transition when confined to channels of $1\ \mu\text{m}$ square cross section. These data test the universality of finite-size scaling as function of ^3He concentration for 1D crossover. The analysis of these data requires that data measured at fixed concentration be converted to a specific heat at constant chemical potential difference $\phi = \mu_3 - \mu_4$. This is carried out according to a procedure performed for planar mixtures by Kimball and Gasparini. We find that, in the most self-consistent analysis of the data, the mixtures define a separate scaling locus from that of pure ^4He , both above and below T_λ . An analysis whereby the exponent α is forced to have the same universal value—as opposed to the best-fit value—yields a good collapse of all the data. This is achieved, however, at a cost of self-consistency. These results mirror very closely those obtained for finite-size scaling of confined planar mixtures, i.e. for 2D crossover.

Helium-4 becomes superfluid at a temperature $T_\lambda(P, x)$ which depends both on ^3He concentration x and pressure P . According to universality, the same critical exponents and ratios of critical amplitudes should be realized for the transition at any $T_\lambda(P, x)$. The same universality should hold for helium uniformly confined. This was tested for the specific heat by Kimball and Gasparini for 2 Dimensional (2D) crossover with films of 48.3 and 986.9 nm [1]. Data for seven different concentrations were used. The conclusion from these data was that 7 different mixtures do determine a universal locus, however, this locus is different from that of pure ^4He . This difficulty could be traced to the fact that the bulk specific heat data, away from $T_\lambda(P_{\text{Sat}}, 0)$, yield a different value for the critical exponent α . In this paper we discuss universality of data taken for 1D crossover as realized with channels of $1\ \mu\text{m}^2$ cross section and 4 mm length. These were formed using a combination of lithography and direct silicon-wafer bonding [2; 3].

Data for both pure ^4He and 2 mixtures were obtained. To analyze the mixture data one must convert the data measured at constant concentration to that at constant chemical potential difference $\phi = \mu_3 - \mu_4$. This was done following a procedure outlined in reference [1]. The bulk specific heat, corresponding to the two mixtures for which data were obtained, $x = 0.1485, 0.2195$, was calculated from existing bulk data [4; 5]. These data were parametrized and interpolated to the desired concentrations. In figure 1 we show the converted data as $C_{P\phi}$ for $x = 0$ and the two concentrations. The behavior of the bulk data for $x = 0$ and both

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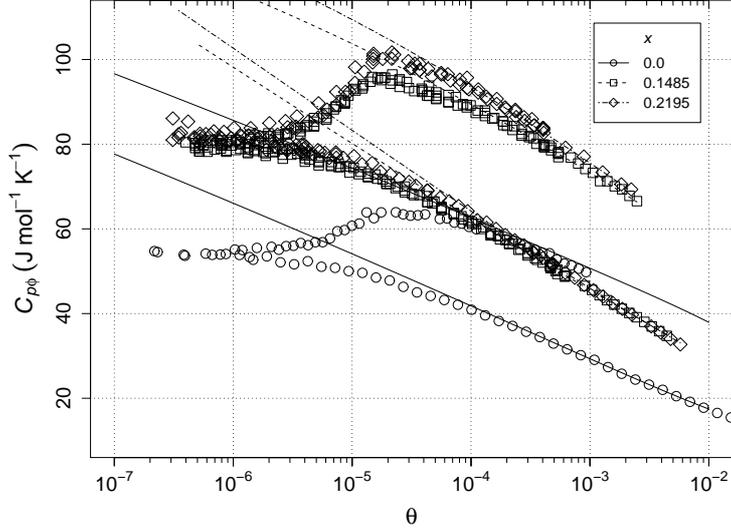


Figure 1. Data converted from the measured specific heat at constant ${}^3\text{He}$ - ${}^4\text{He}$ concentration to a specific heat at constant chemical potential difference between ${}^3\text{He}$ and ${}^4\text{He}$ $C_{P\phi}$. The variable θ is the dimensionless temperature difference from T_λ appropriate for $C_{P\phi}$. The bulk data is shown as lines.

concentrations are shown as the lines. The temperature variable is $\theta = |1 - T/T_\lambda|$ along the thermodynamic path of $\phi = \phi_\lambda$.

Finite-size scaling of these data can be tested using the following functional form [1]

$$\Delta C \left| \frac{\alpha_\phi}{A_\phi(x)} \right| \theta^{\alpha_\phi} = g_2 \left(\left[\frac{L}{\xi(x,t)} \right]^{1/\nu} \right), \quad (1)$$

where $\Delta C = C_{P,\phi}(\theta, \infty) - C_{P,\phi}(\theta, L)$, and $t = |1 - T/T_\lambda|$ along the thermodynamic path of constant x . In this expression one needs the non-universal values of the bulk specific heat amplitude $A_\phi(x)$. This is obtained from available data. One also needs for this expression the values of the correlation length for each bulk concentration. This can be obtained from the superfluid density [6] and its relationship to the correlation length [7; 8].

The scaling of the data according to equation 1 for $T > T_\lambda$ is shown in figure 2. One can see that, in a similar manner as observed for 2D crossover (see figure 3 in reference [1]), the data for the mixtures collapse onto themselves, but on a different locus from that of the pure system. Thus, it is not possible to achieve collapse of these data on a universal curve by using the best-fit parameters for each concentration. It is also not possible to collapse these data if one fixes the exponent to a universal value and refits the bulk data to determine the best-fit value for $A_\phi(x)$ using this constraint. One can only achieve collapse of all the data, as shown in figure 3, by fixing the exponent to a universal value, $\alpha = 0.0115$ for this plot, and then using the value for $A_\phi(x)$ that results from the best-fit value obtained when α_ϕ is allowed to vary in the fit of each concentration's bulk data. This procedure, although successful, is obviously not self-consistent. The best-fit value of α_ϕ for the mixtures is closer to $\alpha_\phi = -0.025$ than to $\alpha = -0.0126$ as obtained from the most precise experimental determination at $x = 0$ [9]. This difference has a substantial effect in equation 1 since α appears not only as the power of θ but also as a multiplicative factor, and makes the finite-size data more sensitive to the universality of α than the bulk data themselves. The universality of α and the ratio of specific heat amplitudes

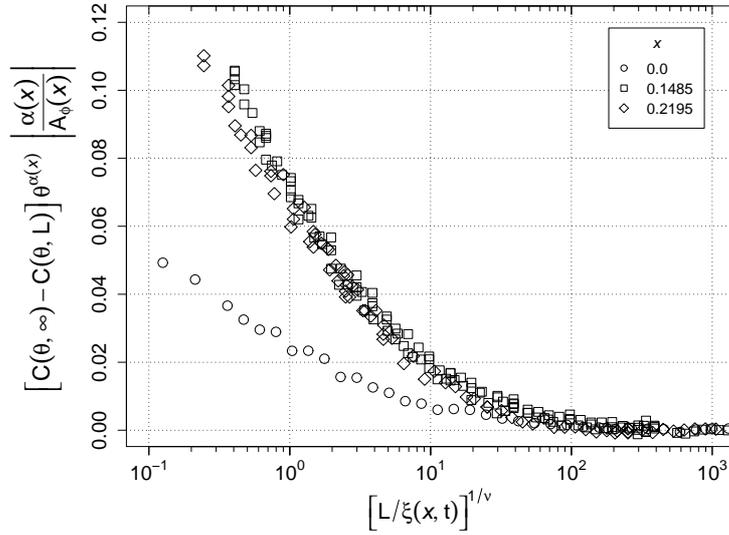


Figure 2. Scaling of the data above the bulk transition temperature T_λ . The data from the mixtures scale well amongst themselves but fail to collapse upon the locus defined by the pure system.

is discussed in more details in reference [10]. The data for $T < T_\lambda$ behave in exactly the same manner as the data for $T > T_\lambda$ and support the same conclusion as above [2]. Thus, it appears from these results, and those of reference [1], that the above behavior is not associated with a particular dimensionality crossover, but is a reflection of the lack of universal behavior of the bulk system. We point out the rather different results reported in another paper for the case of 0D crossover (submitted by Kimball and Gasparini to the LT25). Here it is found that the mixtures do not even collapse on a separate universal locus. It is believed that this rather striking behavior is not a reflection of 0D crossover, but rather that the shallow channels that connect the boxes play an unexpected coupling role in the overall behavior.

Acknowledgments

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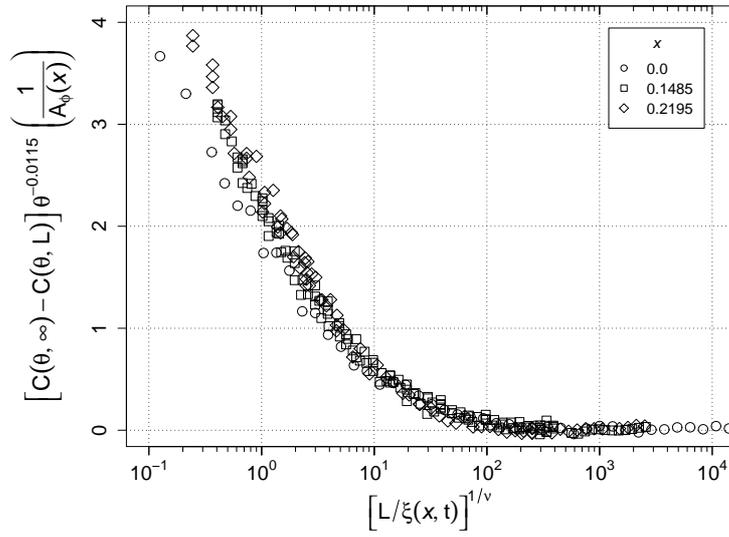


Figure 3. Scaling plot for data above T_λ using a single value of α for all the data rather than the value associated with each one. While this allows the mixture data to collapse with the pure data, this is not a self-consistent analysis as explained in the text.

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