

# Critical Behavior and Scaling of Confined $^3\text{He}$ - $^4\text{He}$ Mixtures.

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*When helium is confined to a uniform small dimension  $L$ , its behavior is modified due to the limitation of the growth of the correlation length. This has been explored for planar confinement over a wide range of  $L$ . Less is known about confined mixtures, especially in the case of the specific heat. Here two principal effects come into play: the magnitude of the correlation length  $\xi$  increases, and the measured heat capacity  $C_{px}$  must be converted to  $C_{p\phi}$ , where  $\phi = \mu_3 - \mu_4$ , before finite-size scaling predictions can be verified. The increase in  $\xi$  makes a given confinement  $L$  “look smaller” as the concentration  $x$  is increased. This, as well as changes of  $L$  itself, can be used to test predictions of correlation-length scaling. We report measurements of the specific heat of confined mixtures and compare with the well established scaling for the pure system.*

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## Introduction

Recent measurements of the specific heat of pure  $^4\text{He}$  confined to a film of thickness  $L$  have shown that one can scale these data as functions of the ratio  $L/\xi$ , where  $\xi$  is the 3D correlation length<sup>1-3</sup>. This can be done everywhere except in the region close to the specific heat maximum and below<sup>1,2</sup>. In the case of mixtures of  $^3\text{He}$  in  $^4\text{He}$ , two effects come into play: the presence of  $^3\text{He}$  renormalizes the thermodynamic response for data taken along paths of constant concentration  $x$ <sup>4,5</sup>; and, the presence of  $^3\text{He}$  increases the correlation length  $\xi$ . This latter effect results<sup>6</sup> in a film of thickness  $L$  looking effectively smaller than at  $x = 0$ . These effects complicate the understanding of the critical behavior of mixtures. However, one expects that a suitable analysis would result in a universal finite-size scaling description no matter

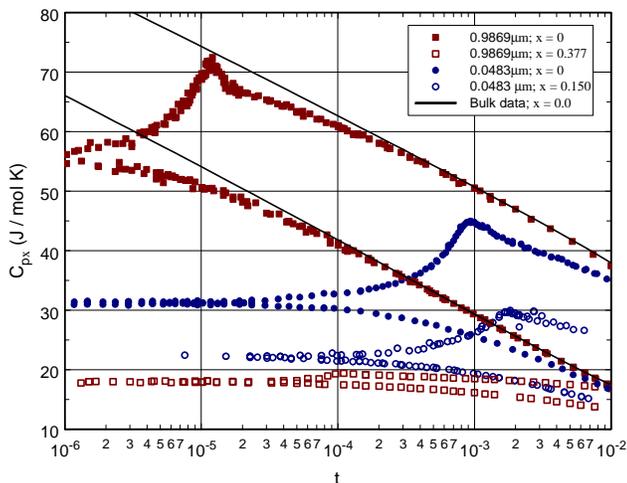


Fig. 1. Heat capacity data taken at constant concentration. For a given confinement, note how the maximum value shifts to a colder temperature as the concentration increases. Also, an increase in  $^3\text{He}$  concentration dramatically reduces the absolute value of the heat capacity.

where and how one crosses the  $\lambda$ -line, or more generally the  $\lambda$ -surface in the  $P$ - $x$ - $T$  space. Thus, the behavior of confined mixtures, much like the behavior of bulk mixtures, may be viewed as a test of *universality* at a second order phase transition. We report in this paper some results for the specific heat of confined mixtures, and the analysis to check on the expected scaling.

### Experimental details and data

Details of the experiment and procedure for the measurements of specific heat have been reported<sup>1,2</sup>. We confine the helium between two silicon wafers which are bonded to achieve a uniform separation. The spacing between the wafers is maintained by a pattern of  $\text{SiO}_2$  posts which are formed lithographically. The bonding is done in a microclean chamber with subsequent annealing at high temperatures. Typical uniformity in the spacing of the wafers is better than 1%. The cell is then staged so that it is weakly linked to two isothermal platforms. One can induce a temperature oscillation in the cell and from the amplitude of the response deduce the heat capacity. The benchmark  $T_\lambda$  of the bulk system is obtained from a small amount of bulk liquid which collects in the filling line.

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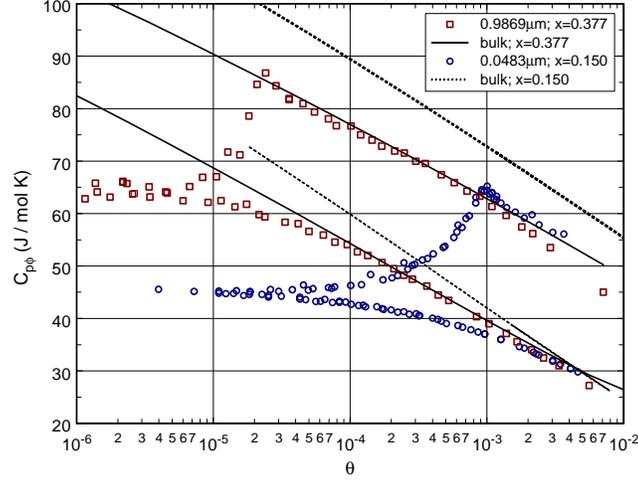


Fig. 2. Heat capacity along paths of constant  $\phi$ . Notice how this plot is remarkably different from  $C_{px}$ , Fig. 1. Also, notice the bulk behavior for the different concentrations. This is later used to scale the data.

Specifically for mixtures, we find that care must be exercised to insure that the concentration remain constant within the cell. We do this by maintaining the isothermal stage of the filling line close to, but slightly warmer than the cell. We also repeatedly determine  $T_\lambda$  of the *bulk* helium and the position of the heat capacity maximum of the *confined* helium over the time that data are taken. Nevertheless, we have seen evidence, especially on the superfluid side, that small concentration differences can be present between the start and end of a data run. These can result, at times, in systematic differences in the heat capacity at a given temperature between different data runs. Thus, our final data for mixtures is the result of multiple runs which were found to be self consistent.

In Fig. 1 are shown the specific heat of helium confined to films of  $L = 0.9869 \mu\text{m}$  and  $0.0483 \mu\text{m}$ . Data for pure  $^4\text{He}^{1,2}$  and  $x = 0.377$  and  $0.150$  respectively are shown. The effect of confinement can be seen by comparing these data to the solid lines, the specific heat of bulk  $^4\text{He}$ . The effect of  $^3\text{He}$  is quite dramatic, depressing the specific heat and producing a shift of the specific heat maximum. To scale these data, one must construct the heat capacity along paths of constant  $\phi$ . This is discussed below.

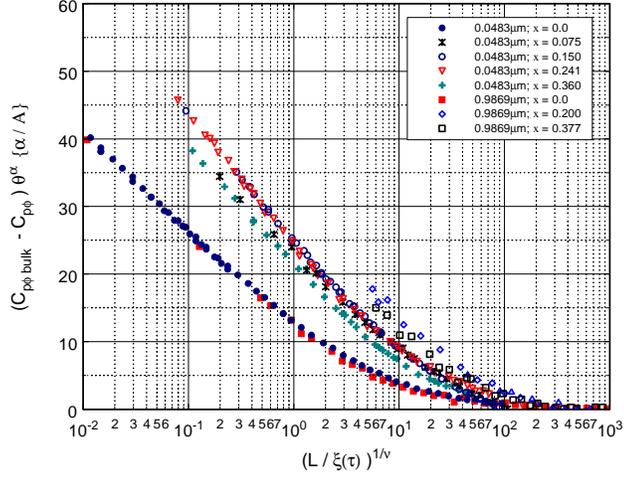


Fig. 3. Scaled data above bulk  $T_\lambda$  with  $\xi = \xi(t)$ .

### Thermodynamic analysis

It is known that a universal behavior of the specific heat (i.e. same critical exponents and amplitude ratios) should be obtained for mixtures only along paths of constant  $\phi^{4,5}$ .  $C_{px}$  can be related to  $C_{p\phi}$  using weakly temperature-dependent derivatives along paths parallel to the  $\lambda$ -line,

$$C_{p\phi} = \frac{C_{px} \left( T \frac{\partial x}{\partial T} \Big|_{p,t} \frac{\partial \phi}{\partial T} \Big|_{p,t} - T \frac{\partial s}{\partial T} \Big|_{p,t} \right) + \left( T \frac{\partial s}{\partial T} \Big|_{p,t} \right)^2}{T \frac{\partial s}{\partial T} \Big|_{p,t} + T \frac{\partial x}{\partial T} \Big|_{p,t} \frac{\partial \phi}{\partial T} \Big|_{p,t} - C_{px}}. \quad (1)$$

Here  $t = |1 - T/T_\lambda(x)|$ . In practice derivatives can be evaluated at  $t = 0$ , and constant  $P$  can be taken as saturated vapor pressure. Eq. 1 makes the conversion between  $C_{px}$  and  $C_{p\phi}$ , but there remains to ascribe to  $C_{p\phi}$  a distance to  $T_\lambda$  along the path  $\phi = \text{constant}$  where the relevant variable is  $\theta = |1 - T/T_\lambda(\phi)|$ . This  $t$  to  $\theta$  conversion requires  $C_{p\phi}^{7,8}$ .

To analyze the present data, we have used bulk data for mixtures<sup>8,9</sup> and interpolated to our concentrations. We have followed a similar procedure for the  $\lambda$ -line derivatives<sup>7,8</sup>. This has enabled us to calculate the bulk  $C_{p\phi}$ , and the variable  $\theta$ . In the case of the confined mixtures, we have followed the same procedure in converting the measured  $C_{px}$  to  $C_{p\phi}$ . This involves the approximation that the derivatives along the  $\lambda$ -line are the same as for the bulk system.

The result of this analysis is shown in Fig. 2. Here we have plotted  $C_{p\phi}$

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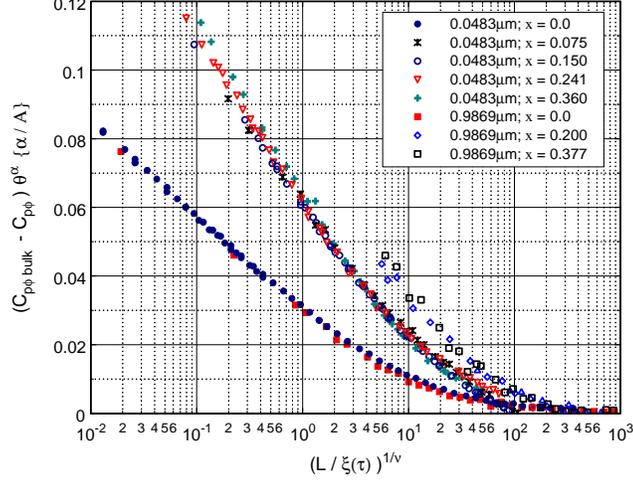


Fig. 4. Data scaled after the specific heat has been divided by the amplitude of the bulk data. The mixture data collapses to a single locus for a single confinement size but different confinements do not collapse together. Also, none of the mixture data scale with the pure system.

for bulk (solid, dashed lines) and two mixtures as function of  $\theta$ . These data are now fully analogous to the data of pure  $^4\text{He}$  shown in Fig. 1. To test correlation-length scaling we construct the following function<sup>1,2</sup>,

$$[C_{p\phi}(\theta, \infty) - C_{p\phi}(\theta, L)]\theta^\alpha = g_2\left(\frac{L}{\xi}\right) = g_2^* \left( \left[ \frac{L}{\frac{m_4^2 k_B T}{4\pi \hbar^2 \rho}} \frac{\rho_s}{\rho} \right]^{1/\nu} \right) = g_2^*(\eta) \quad (2)$$

where one can take  $\alpha = -0.0115$ ,  $\nu = 0.6705$ , and we have written  $\xi$  in terms of  $\rho_s/\rho$ <sup>10-12</sup>. The  $1/\nu$  power is used for convenience. For  $\rho_s/\rho$  we use the parametrization of Schubert and Zimmermann<sup>13</sup>. Since  $\rho_s/\rho$  takes on a unique value at any coordinate  $(t, x)$  (unlike the specific heat where one must specify a path along which the entropy is differentiated), one can evaluate  $\rho_s/\rho$  at the value of  $t$  where the specific heat  $C_{px}$  is measured. The result of this calculation is shown in Fig. 3 for a number of mixtures and  $T > T_\lambda$ . We note first of all that the locus for  $x = 0$  is well defined. While only two values of  $L$  are plotted here, this locus holds for a range of  $L$ 's over a factor of 1000<sup>1-3</sup>. *The mixtures do not collapse on this locus, and do not collapse among themselves.* To remedy this, one must recognize that the amplitude of the bulk specific heat for mixtures changes with concentration. The amplitude is

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never an issue with the pure system since all the specific heats are compared to the same bulk values. To take this variation in the mixtures amplitude into account, we divide the data for a given concentration by the amplitude of the corresponding bulk data. We also do this for the pure system so we may compare it to the mixtures. The result of this procedure is seen in Fig. 4. Notice that now, different mixtures scale well for a single confinement, but different confinements do not scale with each other. Also, none of the mixture data scale with the pure system as one would expect. Perhaps this is not surprising since the bulk mixture data also show non-universal behavior in  $\alpha$  and the amplitude ratios<sup>8</sup>.  $\alpha$  for the mixtures seems to be closer to -0.025 than -0.0115. We note that using this more negative value of  $\alpha$ , and the corresponding  $\nu$ , does not affect our conclusions about the scaling of the confined mixtures.

## Acknowledgment

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