Scaling, Dimensionality Crossover, Surface and Edge Specific Heats in Confined ^{4}He

F. M. Gasparini, M. Diaz-Avila, M. O. Kimball and K. P. Mooney

*Department of Physics, University at Buffalo, State University of New York
Buffalo, NY 14260-1500, USA*

We have studied the superfluid transition of ^{4}He in situations where the growth of the correlation length is limited by a uniform spacial confinement. Under these conditions the critical behavior attained in the thermodynamic limit is greatly modified. One expects that data for similar confinement should scale with the correlation length. This is found to be true for the specific heat in planar confinement, 2D crossover, except in the region of the specific heat maximum and on the superfluid side. The modifications due to confinement depend on the details of the geometry. Studies of the specific heat with 1D and 0D crossover show the important role played by the lower dimension. Far from the transition the details of the confining geometry reveal contributions to the specific heat which can be attributed to surfaces and edges. Comparison of experimental results with theoretical calculations show agreement in some areas and disagreement in others.

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1. INTRODUCTION

The superfluid transition of ^{4}He is greatly affected if the fluid is confined to dimensions which are comparable to the correlation length. If the geometry of confinement is simple and homogeneous, with only one small length scale L, one expects that as L is varied the thermodynamic behavior should scale with the ratio L/ξ.[1] ξ is the temperature-dependent correlation length. This is a situation which is not unique to helium, but applies in general to all second order phase transitions. There are some unique advantages in studying helium. These stem from the fact that the boundary conditions at the confining walls can be specified in a reasonable way, and the fact that the walls do not break the symmetry of the order parameter. It also helps

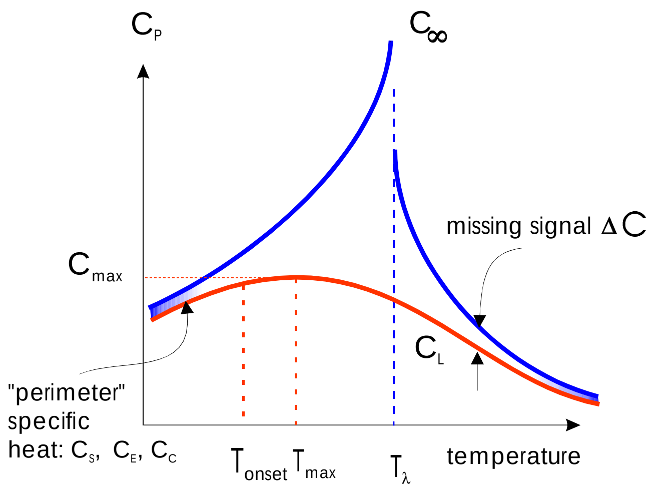

Fig. 1. The specific heat of ^{4}He near the superfluid transition modified due to confinement to a small dimension L .

that the critical behavior of ^{4}He in the thermodynamic limit is well known; hence, deviations from this behavior can be established with high resolution. The superfluid nature of ^{4}He is also useful in realizing confinements where all three spacial dimensions are made small. In Fig. 1 we sketch the specific heat of bulk helium C_∞ and helium confined to a small dimension L, C_L. The most obvious feature of this plot is the rounding of the specific heat with a maximum C_{max} at T_{max} which is shifted below the bulk superfluid transition T_λ. Not obvious from any features in the specific heat is also the fact the superfluid onset is separated from the specific heat maximum and occurs at a lower temperature T_{onset}. A general statement about the overall behavior of C_L is that it represents a missing signal ΔC relative to the bulk behavior. This missing signal is a reflection of the fact that fluctuations near the transition are quenched by the imposition of a spacial constraint. Thus, it is to be expected that if L represents confinement in 1, 2 or all 3 directions the corresponding C_L's would differ. Also identified in Fig. 1 are two shaded regions labeled "perimeter" specific heats. This refers to regions where the correlation length is small relative to L, and deviations from bulk behavior can be attributed to corners, C_c, edges, C_e, surfaces, C_s and even boundary curvature.[2]

The behavior of the confined specific heat can be expressed in several ways reflecting the expectation that as L is changed there should be a scaling of the specific heat with the correlation length. Thus, one may write

$$[C(t,\infty) - C(t,L)]t^{\alpha} = \Delta C t^{\alpha} = g_2(x) \tag{1}$$

where $t = \mid 1 - T/T_\lambda \mid$. The variable x is $tL^{1/\nu}$. The exponents $\alpha = -0.0115$ and $\nu = 0.6705$ characterize the behavior of the bulk specific heat $C_P \sim t^{-\alpha}$, and the correlation length $\xi \sim t^{-\nu}$. Other relationships can be written to describe "single point" aspects of the thermodynamic response such as the shift in specific heat maximum, the value of the specific heat at $t = 0$ or at the maximum, and the shift in the superfluid onset. These relationships are power laws to leading order.

The shaded region in Fig. 1, representing the initial small deviations from bulk behavior, can be described in an approximate manner in such a way that the function g_2 reflects the geometry of the confining surface,[2] thus

$$\Delta C t^{\alpha} = -\frac{g_s A_s}{L\alpha_s} t^{\alpha-\alpha_s} - \frac{g_e A_e}{L^2 \alpha_e} t^{\alpha-\alpha_e} + g'(x). \tag{2}$$

For this equation to be in scaling form one must have $\alpha_s = \alpha + \nu$ and $\alpha_e = \alpha + 2\nu$. The first two terms represent the surface and edge specific heats respectively. Another term involving corners and behaving with the exponent $\alpha_c = \alpha + 3\nu$ could be added if relevant. The function $g'(x)$ represents the behavior of the scaling function once the correlation length becomes comparable to (but still smaller) than L. The amplitudes A_s and A_e are not known and must be calculated theoretically. The terms g_s and g_e are geometric factors reflecting the surface to volume and edge to volume ratios in a particular geometry. Thus as example, for films, channels of square cross section, and boxes the terms are $g_s = 2, 4, 6$ respectively.

The unavoidable dependence of the confined system not only on L but on the shape of the container makes the study of finite-size effects more demanding than studies of bulk criticality. To scale measurements with different L's, say in the case of films, one must maintain the lateral dimensions the same as L is varied. If these lateral dimensions are effectively infinite on the scale of ξ one may also think in this case of a crossover in dimensionality from 3 to 2 as ξ becomes comparable to L . Similarly for long channels and boxes one may think of crossover from 3 to 1 and 0 respectively. Further, it is of interest to look at thin films of thickness L which are also limited in lateral dimension W, thus realizing finite-size effects with dimensionality crossover from 2 to 1.[3]

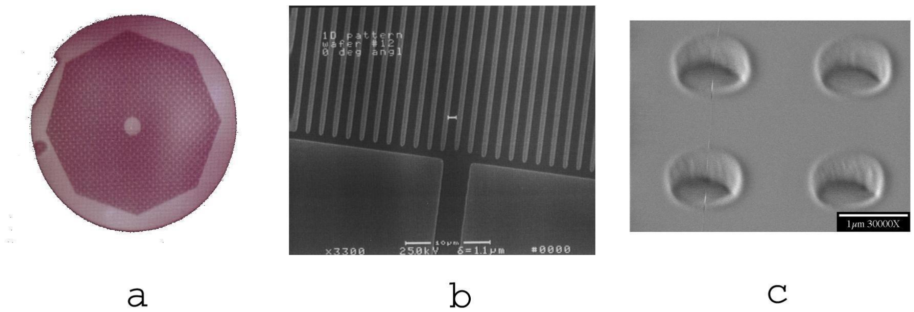

Fig. 2. Examples of various wafers and cell designs.

2. EXPERIMENTAL DETAILS

The key requirement in studies of finite-size effects is the homogeneity of confinement. To achieve this we have developed a technique which combines silicon lithography with direct wafer bonding. Examples of some structures we have made are shown in Fig. 2. Fig. 2a is an infrared image of a 2D cell. It consists of two silicon wafers 5 cm in diameter, separated at 0.3189 μm by an array of silicon dioxide posts— the bright dots in the image. The light border is solid SiO_2 which makes the cell leak tight. There is one imperfection in this cell which is visible as a Newton ring near the perimeter (at eight o'clock). This does not affect the cell since it is within the oxide border. The center region of the cell has a small hole through which helium is introduced. Also for this cell, and not visible in this figure, one has within the small center disk an array of 10 nm$\times$8μm channels. These were designed to study the superfluid fraction and finite-size effects for a laterally confined film.[4]

In Fig. 2b is shown the design of the center region of a cell in which the heat capacity of helium for 1D crossover was studied. This is an electron micrograph showing a 3 μm channel feeding into an array of 1μm$\times$1μm$\times$4mm channels. The lighter shading is SiO_2. Fig. 2c shows 4 pill boxes 1μm in diameter by 1μm in height. These were used to study the heat capacity for 0D crossover.[5] The full cells for these studies contains 0.5 km of channels and 10^9 boxes respectively.

The two wafers when bonded act as a Fabry-Perot interferometer. For cells where the separation between the wafers is larger than 0.5 μm one can examine the homogeneity of the wafer's separation by measuring the interference fringes in the infrared. We find that the separation is homogeneous to better than 1%. Further details about our cells, the lithography, and the bonding process can be found in other publications.[4,6–9]

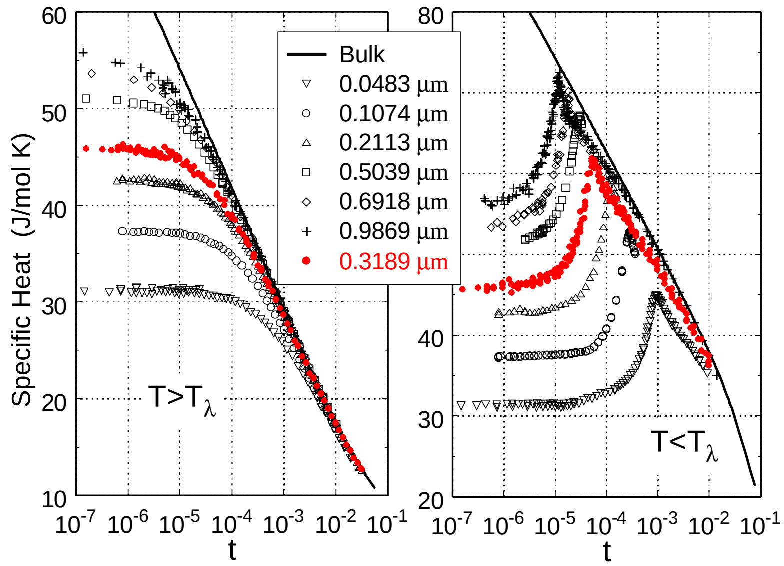

Fig. 3. Specific heat for planar confinement at 7 different L's.

3. EXPERIMENTAL RESULTS

Over the years we have accumulated specific heat data for planar confinement for 7 different spacings. These cells differ in design mostly within the center region near the filling line. These data, along with the latest results for a cell at $L = 0.3189$ μm,[4] are shown in a semilog plot in Fig. 3. The data behave very much as one would expect with deviations from bulk behavior becoming manifest further from the transition the smaller the confinement. Since, as can be seen from Fig. 1, there is no structure at T_λ for the confined helium, these data all approach a constant $C(0, L)$. One finds that $C(0, L) \sim L^{\alpha/\nu}$ as expected, however, the magnitude of $C(0, L)$ is smaller than predicted theoretically.[7] As one moves away from T_λ, the data join the bulk behavior. The data for $T < T_\lambda$ have more structure because of the specific heat maximum which shifts progressively to lower temperatures and becomes smaller as L is decreased. To scale these data, one has to plot them according to Eq. 1. This is shown in Fig. 4 for the branch $T > T_\lambda$.

One can see from Fig. 4 that all of the data which in Fig. 3 occupied different loci have now collapsed on a universal curve. This is the locus of the scaling function $g_2(x)$ for planar confinement. In producing Fig. 4 one has to take the difference between the bulk specific heat and the confined specific heat. When this difference is small it is easy to introduce systematic deviations in the data from this subtraction. This has to do with the choice

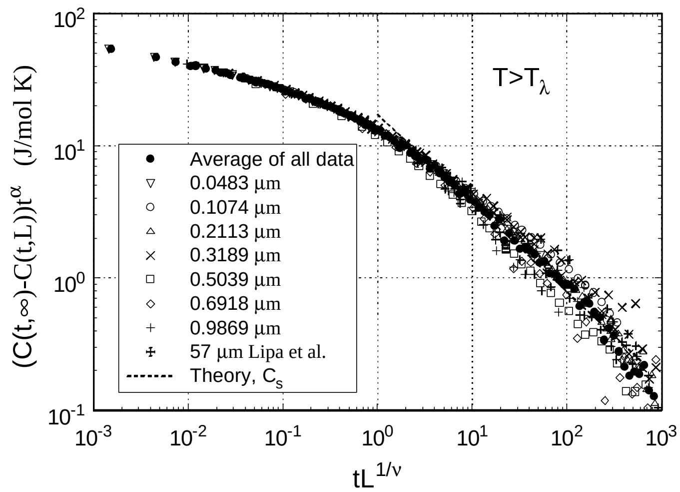

Fig. 4. Scaling of the data above the transition.

of bulk specific heat and the way the confined data are normalized to the bulk behavior for large values of t.[6] We use a combination of data from different investigators to represent the bulk. This gives us the widest range of reliable data in the neighborhood of T_λ including the most important region far from T_λ where the confined data are normalized. We note that for all the data the random errors are smaller than the systematic errors which can be introduced in this analysis. The average value of all these data are shown as solid circles in Fig. 4. We have also included in Fig. 4 the data at 57 μm from the CHEX experiment.[10] All of these data can be compared with the theoretical calculation of the amplitude of the surface specific heat A_s.[11] This is the dashed line in Fig. 4. One can see that the agreement with the average of the data is excellent. We point out that there are no adjustable parameters in this comparison, and that all the ingredients in obtaining A_s come from bulk properties. There is nothing in A_s which refers to silicon confinement or the specific design of the experimental cells. This result is also independent of geometry such as channels or boxes, it is the surface specific heat amplitude for any arbitrary geometry: different geometries would simply bring in different g_s factors. Note that for the planar cells there is no edge specific heat and only the first term in Eq. 2 is relevant. Note also that the surface term dominates only over a limited region of the scaling variable, then gives way to the full function $g'(x)$.

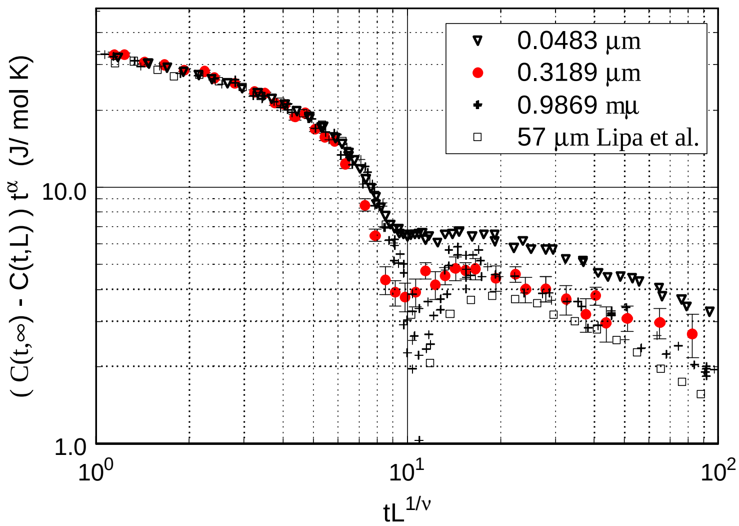

Fig. 5. Lack of scaling near and below the specific heat maximum.

If one compares the data for smaller values of the scaling variable with theory one finds that theories[12,13] underestimate the effects of confinement. A recent calculation[14] however, finds good agreement for all the region $T > T_\lambda$. The disagreement with theory becomes more serious as the specific heat maximum is approached and one goes into the superfluid side. Four sets of data for the superfluid side are shown in a scaling plot in Fig. 5. Unlike the normal side, the superfluid side presents a number of puzzles. One finds that the specific heat scales well for the region close to T_λ, up to $tL^{1/\nu} \sim 6$ (L is in Å) but fails to collapse as one approaches the specific heat maximum and proceeds into the region where the superfluid fraction of the confined helium is not zero. This behavior is consistent with that of the superfluid fraction for 2D confinement which also fails to scale.[15] It is interesting to note that the position of the specific heat maximum, i.e. the shift in transition temperature with L *does scale*. This over 7 decades in L.[7] Thus, it is the *magnitude* of the specific heat and the "shape" of the data which are responsible for the lack of scaling near the maximum. This point is emphasized in Fig. 6 where the following scaling relation is tested just at the maximum,

$$[C(t, L) - C(t_0, L)]L^{-\alpha/\nu} = f_1(x). \quad (3)$$

In the above, t_0 is the temperature at which the correlation length becomes equal to L (this of course never happens in the confined system, but is simply a fixed point which can be considered for purpose of scaling). The scaling function $f_1(x)$ is different from $g_2(x)$ but can be related to it.[7] From Eq. 3 one expects that at $t = t_{max}$ the data should collapse on a constant

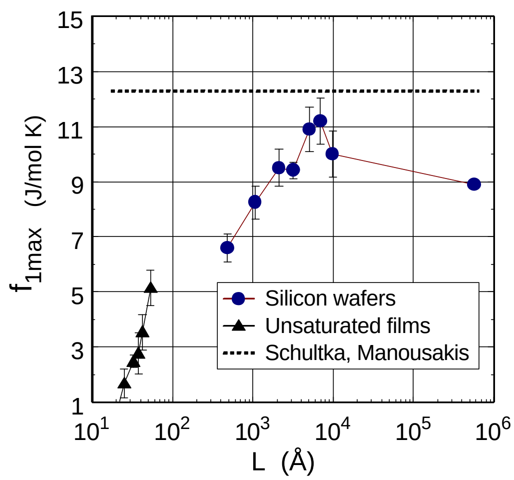

Fig. 6. The value of $f_1(x_{max})$. All the data are expected to collapse on a same value with no variations as function of L. The data at 57 μm are from Ref. 10.

value $f_1(x_{max})$. Instead, Fig. 6 shows a *systematic variation of this value as a function of* L. Interestingly, this trend extends to unsaturated films of relatively modest thickness, $\sim$ 20 to 60 Å. It is not the disagreement with the Monte Carlo calculation,[13] the dotted line in this figure, which is significant in this plot, but rather the systematic variation with L which indicates lack of scaling.

One may ask whether the lack of scaling on the superfluid side is a manifestation of the 2D crossover for films, in which case non-universal behavior might be expected;[16] or, is the lack of scaling a reflection of the fact that the order parameter is not zero, or about to become non-zero? To resolve these issues one must look at confinement with different dimensionality crossover. There are some data for 1D crossover. These data are not as extensive as for 2D, but they do suggest that a similar problem might exist in this case, with a lack of scaling near the heat capacity maximum.[8,17] However, the evidence is not as compelling as for 2D, and further work is necessary.

We have recently obtained data for confinement in silicon structures with 1D crossover and 0D crossover (see Fig. 2 for cell designs). These data are all for the same small dimension $L = 1\mu m$. While these data do not yet answer the questions posed above, they do make an interesting comparison on the role of the lower dimension in the confined system. These data along with the data for 2D crossover are shown in Fig. 7. One can see from this plot that, having fixed L, the lower dimension now plays a strong

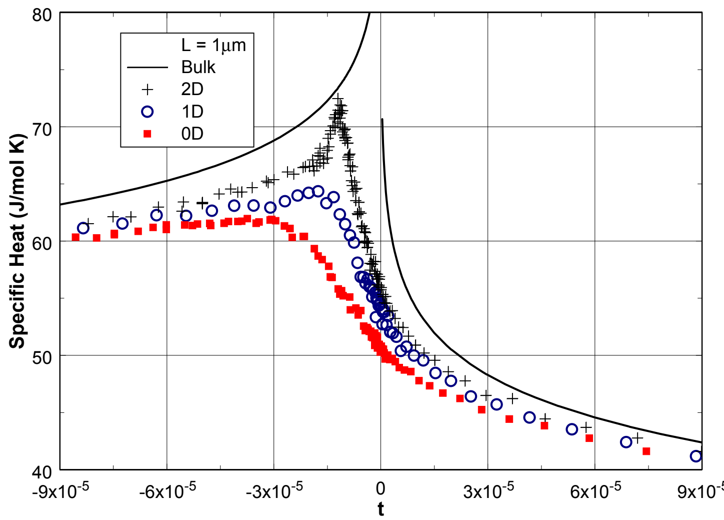

Fig. 7. Specific heat for different dimensionalities crossover.

role in determining the specific heat. There is a progressive decrease in the specific heat as the lower dimension in decreased from that of a film to that of a box. This is consistent with the fact that fluctuations are more effectively quenched as additional spacial dimensions are made small. Note the somewhat unusual shape of the specific heat maximum for the film relative to the other geometries. It is larger, but also seems to be unusually "peaked". This is clearly indicative that these data should scale (or not scale) quite differently from those of the other confinements. These data are discussed in greater details in another paper in these proceedings.[8] In particular, we note that the scaling functions for the boxes and channel yield for the first time a magnitude for the line specific heat (see Eq. 2).[18]

4. SUMMARY

The behavior of the specific heat of confined ^{4}He at the superfluid transition has been studied to test predictions of finite-size scaling and dimensionality crossover. One finds that there is very good data collapse for $T > T_\lambda$ and excellent quantitative agreement with theoretical calculations in the surface specific heat region. However, near the specific heat maximum and at lower temperatures the data fail to collapse. This is a reflection of the magnitude of the specific heat rather than the shift in the specific heat maximum, which in fact does scale. Data for L=1μm but different geometric confinements show the role of the different lower dimensions. These data also yield for the first time in a critical system the edge contribution to the critical behavior. Two major results of this work are the confirmation of correlation-length scaling for $T > T_\lambda$ and the observation of lack of scaling for $T < T_\lambda$. It remains to be established whether this is limited to 2D crossover or a general result for all lower dimensions.

5. ACKNOWLEDGMENTS

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